

Lattice QCD with Domain-Wall Fermions *

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We study the quenched lattice QCD using domain-wall fermions at $\beta = 6.0$. Behaviors of both pion mass and the explicit breaking term in the axial Ward-Takahashi identity support the existence of the chiral zero modes. We observe a good agreement between the pion decay constants f_π from both the conserved axial current and the local current perturbatively renormalized at 1-loop. Finally the possible existence of the parity broken phase is also examined in this model.

1. Introduction

Domain-wall QCD (DWQCD)[1, 2] is considered to have good properties such as exact chiral symmetry without doublers, no $O(a)$ scaling violation and the existence of conserved axial current. There exist several pilot studies[2], which seems to support these superior properties.

We study DWQCD in the quenched approximation. First we investigate the pion mass and the explicit breaking term of the axial Ward-Takahashi identity to confirm the existence of the chiral zero mode at zero current quark mass. Next we calculate the pion decay constant f_π from both the conserved axial current and the local axial current, using the perturbative renormalization factor[4] for the latter, in order to check the reliability of the lattice perturbation theory. Finally we explore negative m_f region to examine the existence of the parity broken phase predicted in [3].

2. Chiral symmetry

The fermion action is identical to the original one[1], with domain wall height M , bare quark mass m_f and the extent of the 5th dimension N_s . We employ 10–30 gauge configurations, generated by the plaquette action at $\beta = 6.0$ ($a^{-1} \sim 2$ GeV) on $16^3 \times 32 \times N_s$ lattices. The unit wall source without gauge fixing is used for quark propagators. The mean-field estimate for the optimal value of M , $M = 1 + 4*(1-u)$ with $u = 1/(8K_c)$, gives $M = 1.819$, from $K_c = 0.1572$ for the Wilson fermion at $\beta = 6.0$.

First we investigate the existence of the chiral zero mode in the chiral limit of the model,

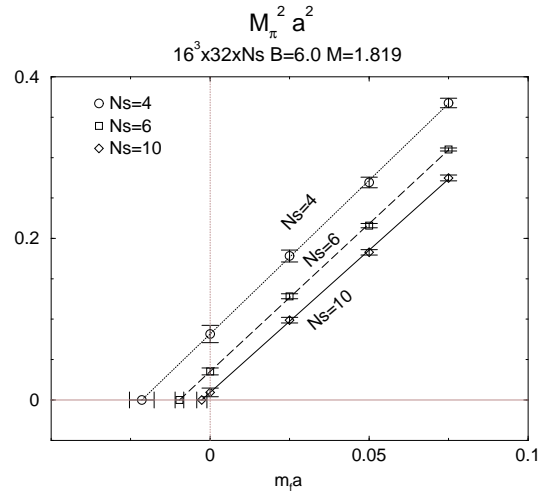


Figure 1. Pion mass squared as a function of $m_f a$ at $M = 1.819$ and $N_s = 4, 6, 10$. Solid lines show linear fits.

$m_f \rightarrow 0$ and $N_s \rightarrow \infty$. The pion mass squared is plotted as a function of $m_f a$ in Fig.1. Since the linearity of M_π^2 in m_f is well satisfied, we linearly extrapolate it to $m_f = 0$ for each N_s . We also evaluate a critical quark mass $m_c(N_s, M)$ at which the pion mass squared vanishes.

In Fig. 2, M_π^2 is plotted as a function of N_s for $m_f a = 0.025, 0.050, 0.075$ and $\rightarrow 0$. Extrapolated values of M_π^2 at $m_f = 0$ seem to vanish exponentially in N_s , while M_π^2 at finite m_f remains non-zero. For $N_s = 10$, $M_\pi^2(m_f = 0)$ is already as small as that for the NG pion of the KS fermion at the same β for the same spatial lattice size. Furthermore $m_c(N_s, M = 1.819)$ and the WI-mass[1], defined by $\langle J_5^q | P \rangle / \langle P | P \rangle$, also decrease exponentially in N_s , as shown in Fig. 3. All these facts indicate that the chiral symmetry is restored for $m_f \rightarrow 0$ and $N_s \rightarrow \infty$ at $\beta = 6.0$.

The lattice scale a is set by the ρ meson mass.

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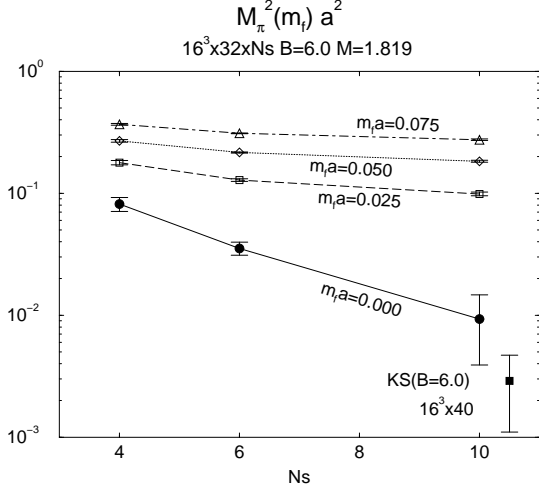


Figure 2. Pion mass squared as a function of N_s at $M = 1.819$. Filled square shows the value for the NG pion of the KS fermion at $\beta = 6.0$ on a $16^3 \times 40$ lattice.

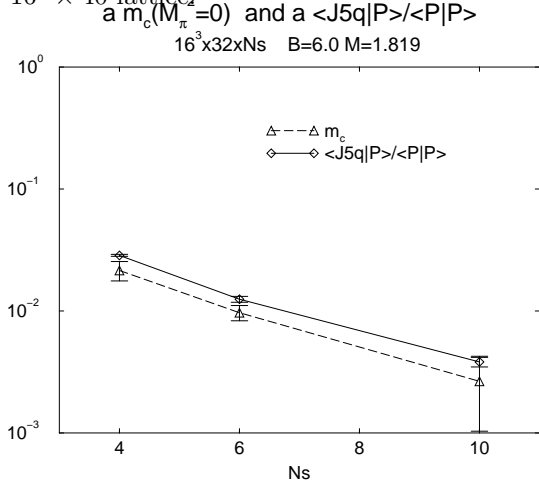


Figure 3. critical quark mass m_c and WI-mass $\langle J_5^0 | P \rangle / \langle P | P \rangle$ extrapolated to $m_f = 0$ as a function of N_s at $M = 1.819$.

In Fig. 4, $M_\rho a$ is plotted as a function of N_s . Circles show linearly extrapolated values to $m_f = 0$, while squares show those to $m_f = m_c$. For small $N_s (= 4)$ the two ways of extrapolation give different results, while two extrapolations are almost identical to each other for $N_s = 10$. We also find that the dependence of M_ρ on the domain-wall height is mild for $M = 1.7-1.9$ and $N_s = 10$: within statistical errors the extrapolated values are consistent with each other.

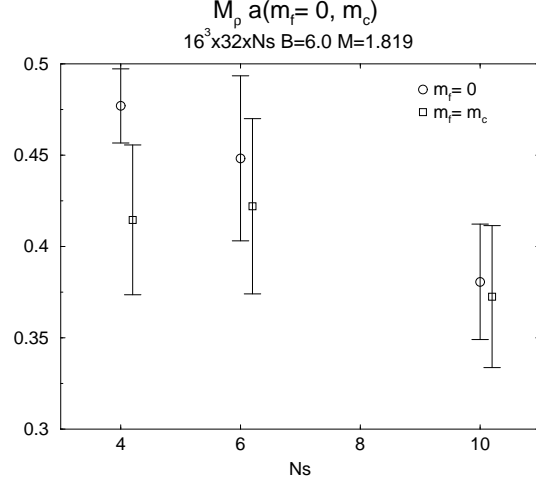


Figure 4. Rho meson mass extrapolated to $m_f = 0$ and $m_f = m_c$ as a function of N_s at $M = 1.819$.

3. Pion decay constant f_π

The pion decay constant is defined as $m_\pi f_\pi / Z_A = \langle 0 | A_4 | \pi \rangle$, which is obtained from correlation functions of pseudo scalar density $P(t)$ and axial current $A_\mu(t)$ at zero momentum: $\langle X(t) Y(0) \rangle = C_{XY} G(t)$ with $G(t) = \exp(-M_\pi t) / (2M_\pi V_s)$ for $X, Y = P, A_4$. For the renormalization factor for the axial current Z_A , we take unity for the conserved current and use the value from the mean-field improved perturbation theory [4] for the local current.

In Fig. 5, f_π is shown as a function of m_f . Circles are obtained from C_{AP} for the conserved axial current, while squares from C_{AP} and diamonds from C_{AA} for the local axial current. The triangle is the experimental value. For local current, filled(open) symbols represent values with(without) perturbative corrections. Three different estimates reasonably agree for all m_f within current statistics, if 1-loop corrections are included. It is also noted that the value of f_π from the conserved current, linearly extrapolated at $m_f = 0$, is close to the experimental value.

4. Parity broken phase

For finite N_s , the parity broken phase may exist in negative m_f regions. As N_s increases, the broken phase shrinks rapidly, and the phase bound-

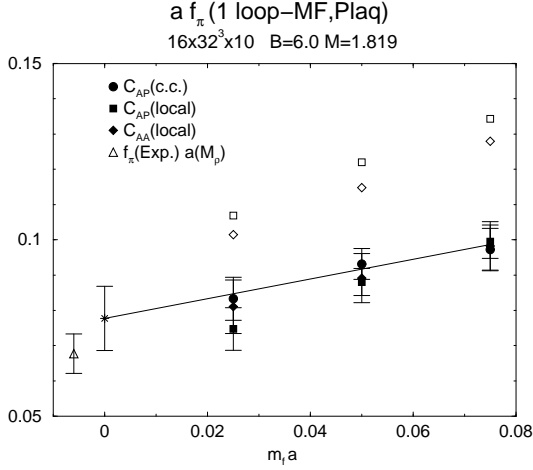


Figure 5. f_π in lattice unit as a function of $m_f a$ at $M = 1.819$ and $N_s = 10$, together with the experimental value.

ary, where the pion mass vanishes, converges to $m_f = 0$ [3]. To examine this parity broken picture in DWQCD, we calculate pion masses at $m_f a = -0.120, -0.100, -0.080$ for $N_s = 4$ and $M = 1.819$. The pion propagators at these parameters form peculiar shapes similar to “W” character, which has been often observed for the Wilson fermion near or in the parity broken phase. Pion mass squared as a function of m_f is shown in Fig. 6. Extrapolations of M_π^2 to zero both from positive and negative m_f (two largest m_f are used for negative m_f) indicate that a parity broken phase may exist around $m_f a \sim -0.03$ at this parameter. Needless to say, more high statistics and variation of parameters are necessary for the definite conclusion.

5. Conclusions and discussions

At $\beta = 6.0$ we have several indications that the chiral symmetry is restored at $N_s \rightarrow \infty$.

We see that all three different estimates for f_π are consistent with each other. This shows the mean-field improved perturbation theory works well at $\beta = 6.0$ in DWQCD. Although the value of f_π at $\beta = 6.0$ turns out to be compatible to

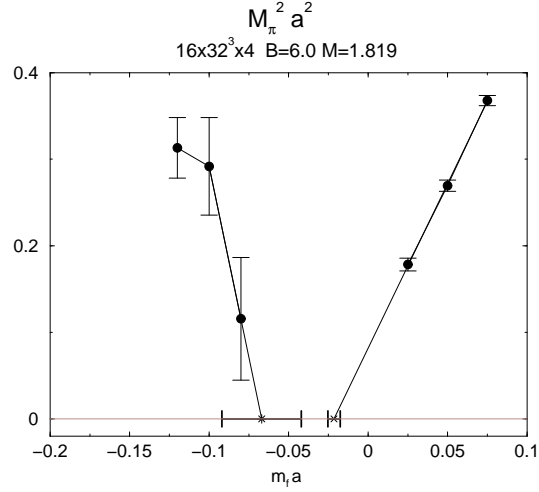


Figure 6. M_π^2 in lattice unit as a function of $m_f a$ at $M = 1.819$ and $N_s = 4$.

the experimental one, detailed scaling studies at different β 's are needed before making the firm statement. The chiral symmetry in DWQCD, however, may fail to recover on the coarse lattice [5]. If this is true, the scaling studies of DWQCD become rather difficult.

We examine negative m_f to see the parity broken phase in DWQCD. The result seems consistent with the parity broken picture, though further confirmations for this are definitely required.

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